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IMPROVED "THREE-COMPONENT" THEORY OF NUCLEATE POOL BOILING

J. MADEJSKI
Instytut Badań Jadrowych, Świerk k/Otwocka, Poland

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Abstract—Previously proposed "three-component" boiling heat transfer theory is improved in this paper. The three components of the heat flux are: the latent heat transport rate; the molecular heat conduction rate; and the turbulent convection in liquid. In contrary to the previous study the heat transfer equation is assumed to be valid in the boundary layer also. The general result of integration and its special simplified forms are discussed. One of these primitive forms is deduced in a simplified manner taking into account the temperature gradient at the wall. This form contains only one empirical constant of the order 0·12–0·18 for water and liquid metals.

NOMENCLATURE

a',	thermal diffusivity of the liquid;
$\lvert Ar \rvert$,	Archimedes modulus;
B,	dimensionless parameter, equation (2.26);
c'_p,	specific heat of the liquid at constant pressure;
C,	dimensionless parameter, equation (2.26);
C_0,	constant, equation (2.36);
C_j,	constant, equation (2.7);
C_n,	constant, equation (2.2);
C_t,	constant, equation (2.6);
C_w,	constant, equation (1.7);
C_α,	constant, equation (1.6);
C_β,	constant, equation (2.7);
D,	local bubble diameter;
D_c,	initial bubble diameter;
$\tilde{D}$,	dimensionless bubble diameter D/D_c;
f,	frequency of bubble emission;
g,	acceleration due to gravity;
Δh,	latent heat of vapourization;
$\lvert Ja \rvert$,	Jakob number;
$\lvert Ku \rvert$,	Kutateladze number;
l,	characteristic dimension of the heat-transfer process, equation (1.7);
m,	dimensionless parameter, equation (2.27);
M,	a function, equation (2.28);
n,	number of bubble columns per unit area;
n_i,	population of cavities of radius R_i;
n_c,	entire population of cavities;
$\lvert Nu \rvert$,	Nusselt number;
p,	pressure;
p'_s,	$(\mathrm{d}p/\mathrm{d}T)_{T=T_s}$;
q,	heat flux;
q', q'', q''',	components of the heat flux due to flow of bubbles, to conduction, and to eddy convection, respectively;
R_1,	maximum roughness of the surface;
R_{ms},	mean-square roughness of the surface;
R_i,	radius of a cavity;
$\lvert Re \rvert$,	Reynolds number for boiling heat transfer;
t,	time;
T,	mean temperature of the liquid;
T_s,	saturation temperature;
ΔT,	difference between the temperature of the surface and the saturation temperature;
U,	dimensionless parameter, equation (2.9);
v_1, v_2, v_3,	dimensionless parameters, equation (2.10);

w,	relative bubble velocity;
x,	dimensionless parameter, equation (2.24);
y,	coordinate normal to the heated surface.

Greek letters

α,	heat-transfer coefficient for nucleate boiling;
α_D,	heat-transfer coefficient for moving bubbles;
β,	contact angle;
ε,	eddy diffusivity;
η,	quantity defined by equation (2.21);
ϑ,	temperature difference between liquid and vapour;
λ',	thermal conductivity of the liquid;
ν',	kinematic viscosity of the liquid.
ρ', ρ'',	mass densities of the liquid and of the vapour, respectively;
$\Delta\rho$,	$\rho' - \rho''$;
σ,	surface tension.

1. THE MECHANISM OF NUCLEATE POOL BOILING

ONE OF the crucial problems in pool boiling study is the nucleation. The state of art in this field is precisely described by the following comment due to Rohsenow ([1], p. 177): "As yet there is no completely satisfactory theory of nucleation. Most of the proposed theories assume the existence of "nuclei" from which bubbles originate, but the exact nature and origin of these nuclei remains a mystery".

To throw light on this matter much experimental and theoretical work is needed, and a lot of scientific discussion. The following lines, containing some novel ideas, are addressed to investigators in pool boiling area, and express the personal opinion of the present author open for discussion.

The first problem to be discussed here is the nature of nuclei causing the incipience of pool boiling. Only one type of nuclei, namely small amounts of gas entrapped in microscopic crevices in a solid surface of the heated wall, will be taken into account. This type of nuclei is the most important, because, firstly, the deposits desorbed from contaminated liquid form also cavities on the heated wall and, secondly, the desorption of dissolved gas, contained in the liquid, takes place near the heated surfaces, and this gas may be also entrapped by natural or new formed cavities.

Every metallic surface used as a heater in pool boiling has cavities, the dimensions of which may be roughly determined by the radius R_i of a sphere of the same volume as the cavity. The dimension R_i is a statistic variable in such a sense that on a unit area there are n_1 cavities of radius R_1, n_2 cavities of radius $R_2 < R_1$, etc. and n_i cavities of radius R_i. The numbers n_i are populations of cavities of dimension R_i, and a somewhat rough estimation yields

$$\sum_{i=1}^{i=j} n_i R_j^2 = 1, \qquad (1.1)$$

where R_1 is the dimension of the greatest cavity, and R_j that of the smallest. Since R_j must not be zero the number of cavities must be finite, and

$$\sum_{i=1}^{i=j} n_i = n_c. \qquad (1.2)$$

The mean-square dimension R_{ms} is defined by

$$R_{ms} = \left(\sum_{i=1}^{i=j} n_i R_i^2 \Big/ \sum_{i=1}^{i=j} n_i \right)^{\frac{1}{2}} = n_c^{-\frac{1}{2}} \qquad (1.3)$$

and may be treated as the mean-square roughness of the machined surface. Therefore the value of R_{ms} is measurable for each clean surface and is unvariable in the boiling process under the condition that the boiling liquid is not contaminated. The whole finite number of cavities or potential nuclei is thus defined by

$$n_c = 1/R_{ms}^2. \qquad (1.4)$$

Before filling of the apparatus with liquid, the metallic surface is in contact with air, which naturally must be trapped in the cavities. Thus the possibility of bubble nuclei is created before the contact of liquid with the wall. In the process of wall heating the air in cavities becomes mixed with vapour. The radius of such gas–vapour nuclei is naturally determined by the radius of cavity, and their statistics is that of the statistics of the surface roughness. In particular, the radius of the greatest nucleus is of the order R_1, the dimension of the greatest cavity. This radius determines the incipience of pool boiling with gradually increased wall superheat and heat flux. Therefore knowing R_1 and the relationship $q/\Delta T$ for natural convection the conditions of incipient pool boiling may be estimated [2].

After the activation of definite nucleus of radius R_i and emission of the first bubble the dimension R_i becomes unimportant, because the taking-off bubble leaves on the nucleation site after departure a very little vapour bubble of radius evidently far greater than R_i. This small bubble begins to grow up in the first moment, but the colder liquid which is pushed against the wall by the departed bubble causes soon that the equilibrium value of the little bubble is too small. Therefore this bubble begins to condense. This process is connected with the so-called waiting period, during which due to wall heating the equilibrium between liquid and bubble is established. This allows the small bubble to grow up to the departure dimensions.

The described phenomenon is very important, because it follows that the initial bubble diameter D_c at the beginning of the working period is determined not by the dimension of the cavity, but by the wall superheat and actual heat flux.

At low heat fluxes, i.e. in the discrete bubbles region, each activated site produces bubbles with frequency f. These bubbles are separated and form a chain over each activated nucleus. Therefore in this region the number of bubble columns is equal to the number of activated nuclei. At higher heat fluxes, however, the phenomenon of transverse coalescence is observed. Because of greater population of nuclei the groups of growing bubbles agglomerate, and thus one big mushroom-bubble is situated on several activated nuclei. In this region the number of bubble columns n is far smaller than the number of nuclei. This statement is important because there is a confusion between the population of active nuclei and the number of bubble columns per unit area. Both quantities are equal only in the region of discrete bubbles, which covers only a small portion of nucleate boiling curve. On the contrary mushroom-bubbles are present essentially up to the critical conditions. The number of vapour paths in this case is much smaller than the population of nuclei, and the former evidently varies much slower than the latter.

The results of experiments indicate that vapour bubble grows, sometimes very intensely, during the flow through slightly superheated bulk liquid. For this process mainly strong agitation of liquid by the motion of bubbles is reponsible. Heat transfer causing evaporation which may be analyzed in the same way as heat convection by use of equation of Bošnjaković

$$\frac{\mathrm{d}D}{\mathrm{d}t} = \frac{2\alpha_D(T - T_s)}{\rho''\Delta h}, \tag{1.5}$$

where D is the local bubble diameter, ρ'' is vapour density, Δh is heat of vapourization, T is local liquid temperature, T_s is saturation temperature, and α_D is the heat transfer coefficient. Since vapour bubbles behave approximately like solid bodies, the heat transfer coefficient may be calculated from the formula

$$\alpha_D = C_\alpha \frac{\lambda'}{D} |Pr|^{\frac{1}{3}} \left(\frac{wD}{\nu'}\right)^s, \quad |Pr| = \nu'/a', \tag{1.6}$$

where

$$w = C_w \left(\frac{gl\Delta\rho}{\rho'}\right)^{\frac{1}{2}}, \quad C_w = 1{\cdot}18, \quad l = \left(\frac{\sigma}{g\Delta\rho}\right)^{\frac{1}{2}}, \tag{1.7}$$

and w denotes the bubble velocity according to Peebles and Garber. These equations have been utilized in [3] for the analysis of temperature distribution in the bulk liquid, where it has been assumed that the temperature T is quasistationary. It seems that such an assumption is reasonable for the bulk liquid, whereas in the boundary layer the temperature fluctuations are much more distinct. Therefore in [3] the analysis has been restricted to the bulk liquid, and that procedure involves introduction of several empirical constants the nature of which is not simple.

In this paper an attempt to extend the analysis presented in [3] into the boundary layer is done. It is thus assumed that equations (1.5)–(1.7) hold also in the boundary layer. As the initial bubble diameter at $y = 0$ (at the wall) the aforementioned diameter D_c is assumed. Further details of the theory are similar to those presented in [3], although the results obtained are somewhat different and involve a smaller number of empirical constants. Also, in the present paper the heat conduction, particularly important in the boundary layer, has been taken into account in contrary to the theory in [3].

Although there are several other possible criteria of the nucleate boiling crisis, the previously proposed idea of transverse bubble coalescence may be utilized here similarly as in [3]. This criterion is different from the criterion of vapour blanketing of the surface (Rohsenow). Surely the entire blanketing is not needed to cause nucleate boiling crisis, because flowing bubbles grow considerably in bulk liquid, and once coalesced at some distance from the wall they break off the liquid delivery to the wall which consequences in boiling crisis.

2. MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

Similarly as in [3] the heat flux q is assumed to consist of three components, namely (1) latent heat transport rate q', (2) molecular heat conduction in liquid, q'', and (3) turbulent convection in liquid due to agitation, q'''. These components are expressed by

$$\left.\begin{aligned} q' &= \frac{\pi}{6}\rho''\Delta h D^3 nf, \\ q'' &= -\lambda'\frac{\mathrm{d}T}{\mathrm{d}y}, \\ q''' &= -\varepsilon\rho' c'_p\frac{\mathrm{d}T}{\mathrm{d}y}. \end{aligned}\right\} \quad (2.1)$$

Using the expression

$$\frac{\mathrm{d}T}{\mathrm{d}y} = \frac{\mathrm{d}\vartheta}{\mathrm{d}D}\frac{\mathrm{d}D}{\mathrm{d}t}\frac{1}{\left(1 - \frac{\pi n}{4C_n}D^2\right)w}, \quad (2.2)$$

derived in [3], and the equation (1.6), one obtains

$$\frac{\mathrm{d}T}{\mathrm{d}y} = \frac{\alpha_D}{\rho''\Delta hw\left[1 - (\pi n/4C_n)D^2\right]}\cdot\frac{\mathrm{d}\vartheta^2}{\mathrm{d}D}, \quad (2.3)$$

where

$$\vartheta = T - T_s. \quad (2.4)$$

The used notations are explained in the list of symbols; it should be emphasized, however, that n denotes the number of bubble columns or vapour paths. Summation of the heat flux components (2.1) with utilization of (2.4) yields the fundamental equation

$$q = \frac{\pi}{6}\rho''\Delta hnf D^3 - \left(1 + \frac{\varepsilon}{a'}\right)\frac{\lambda'\alpha_D}{\rho''\Delta hw\left(1 - \frac{\pi n}{4C_n}D^2\right)}\cdot\frac{\mathrm{d}\vartheta^2}{\mathrm{d}D}, \quad (2.5)$$

the boundary conditions for which are the following. At the wall it is $D = D_c$, and then $\vartheta = \Delta T$. At some distance comparatively far from the wall in bulk liquid the bubbles cease to grow so that $\mathrm{d}\vartheta/\mathrm{d}D = 0$, which should occur at $\vartheta = 0$, whence in this situation it must be $q = q'$.

In paper [3] the influence of the exponent s in equation (1.6) has been discussed in the range $0 < s < 1$. In result negligible influence of this exponent has been found. Therefore in the present analysis the value of $s = 1$, connected with Reynolds' analogy and allowing simple integration of (2.5), has been assumed. The value of eddy diffusivity

$$\varepsilon = C_t\nu'\frac{nD^4}{l^2}|Ar|^{\frac{1}{2}},\ l^2 = \frac{\sigma}{g\Delta\rho}, \quad (2.6)$$

derived in [3], has been also utilized here as well as the expression for frequency of bubble emission

$$f = \frac{\nu'}{l^2}\frac{C_j}{C_\beta}|Ar|^{\frac{1}{2}}, \tag{2.7}$$

with $C_j = 0{\cdot}59$ or $0{\cdot}56\sqrt{C_\beta}$, and $C_\beta = 1{\cdot}2\,\beta$, where β is the contact angle in radians.

After the nondimensionalization of the variables the equation (2.5) takes the following form

$$\left(\frac{v_1}{v_2}\right)^3 = \tilde{D}^3 - \frac{(1/v_2^4) + \tilde{D}^4}{1 - v_3 v_2^2 \tilde{D}^2}\cdot\frac{\mathrm{d}U}{\mathrm{d}\tilde{D}}, \tag{2.8}$$

where

$$\left.\begin{aligned} \tilde{D} &= D/D_c, \\ U &= \frac{6C_t C_\alpha C_\beta}{\pi C_j}|Pr|^{-\frac{2}{3}}|Ja|^2\left(\frac{\vartheta}{\Delta T}\right)^2, \end{aligned}\right\} \tag{2.9}$$

and

$$\left.\begin{aligned} v_1 &= C_t^{\frac{1}{4}}\left(\frac{6C_\beta}{\pi C_j}\right)^{\frac{1}{3}}|Re|^{\frac{1}{3}}|Ar|^{-\frac{1}{24}}|Pr|^{\frac{1}{4}}(nl^2)^{-\frac{1}{12}}, \\ v_2 &= C_t^{\frac{1}{4}}|Ar|^{\frac{1}{8}}|Pr|^{\frac{1}{4}}(nl^2)^{\frac{1}{4}}\frac{D_c}{l}, \\ v_3 &= \frac{\pi}{4C_n C_t^{\frac{1}{2}}}|Ar|^{-\frac{1}{4}}|Pr|^{-\frac{1}{2}}(nl^2)^{\frac{1}{2}}. \end{aligned}\right\} \tag{2.10}$$

The dimensionless moduli are defined as follows

$$\left.\begin{aligned} |Ja| &= \frac{c'_p\rho'\Delta T}{\rho''\Delta h}, \\ |Re| &= \frac{ql}{\rho''\Delta h\nu'}, \\ |Pr| &= \nu'/a', \\ |Ar| &= \frac{g\Delta\rho l^3}{\rho'\nu'^2}. \end{aligned}\right\} \tag{2.11}$$

The boundary conditions are

$$U(1) = U_0 = \frac{6C_t C_\alpha C_\beta}{\pi C_j}|Pr|^{-\frac{2}{3}}|Ja|^2, \tag{2.12}$$

and

$$U\left(\frac{v_1}{v_2}\right) = 0. \tag{2.13}$$

The solution of (2.8) satisfying (2.13) yields for the condition (2.12) the following formula:

$$\begin{aligned} U_0 = {}& \frac{1}{4}\ln\frac{1 + v_2^4}{1 + v_1^4} + \frac{v_1^3}{4\sqrt{2}}\left(\ln\left|\frac{v_1^2 + v_1\sqrt{2} + 1}{v_2^2 + v_2\sqrt{2} + 1}\right.\right. \\ & \times \left.\frac{v_2^2 - v_2\sqrt{2} + 1}{v_1^2 - v_1\sqrt{2} + 1}\right| + 2\,\mathrm{arctg}\frac{v_1\sqrt{2}}{1 - v_1^2} \\ & \left. - 2\,\mathrm{arctg}\frac{v_2\sqrt{2}}{1 - v_2^2}\right) + \frac{v_3}{2}\left[v_1^2 - v_2^2 - \mathrm{arctg}\,v_1^2\right. \\ & + \mathrm{arctg}\,v_2^2 + \frac{v_1^3}{2\sqrt{2}}\left(\ln\left|\frac{v_1^2 + v_1\sqrt{2} + 1}{v_2^2 + v_2\sqrt{2} + 1}\right.\right. \\ & \times \left.\frac{v_2^2 - v_2\sqrt{2} + 1}{v_1^2 - v_1\sqrt{2} + 1}\right| - 2\,\mathrm{arctg}\frac{v_1\sqrt{2}}{1 - v_1^2} \\ & \left.\left. + 2\,\mathrm{arctg}\frac{v_2\sqrt{2}}{1 - v_2^2}\right)\right]. \end{aligned} \tag{2.14}$$

Of practical interest is the case when $v_1 \gg 1$, and simultaneously $v_2 \ll 1$. Then the result (2.14) simplifies to give

$$U_0 \approx v_2 v_1^3 - \tfrac{1}{2}v_3 v_1^2. \tag{2.15}$$

Substitution of (2.10) and (2.12) into (2.15) yields after some rearrangement

$$\begin{aligned} |Ja|^2 = {}& \frac{D_c}{C_\alpha l}|Pr|^{\frac{5}{3}}|Re| \\ & - \frac{\pi}{8C_t C_\alpha C_n}\left(\frac{\pi C_j}{6C_\beta}\right)^{\frac{1}{3}}|Ar|^{-\frac{1}{3}}(nl^2)^{\frac{1}{3}}|Re|^{\frac{2}{3}}|Pr|^{\frac{2}{3}}. \end{aligned} \tag{2.16}$$

The initial bubble diameter D_c is to be evaluated according to actual operating conditions determined by the wall superheat ΔT or the Jakob number $|Ja|$, and the heat flux q, or the boiling Reynolds number $|Re|$. Paper [2] is devoted to this problem, and the relationship between the quantities

$$\frac{\Delta\rho}{\rho}\cdot\frac{p'_s\Delta TD_c}{4\sigma} \text{ and } \frac{\alpha D_c}{2\lambda'}$$

is derived therein. It is possible to assume a very simple approximate relation, namely

$$\frac{\Delta\rho}{\rho}\cdot\frac{p'_s\Delta TD_c}{4\sigma} \approx 1 + \frac{\alpha D_c}{2\lambda'}, \tag{2.17}$$

from which it follows that

$$D_c = \frac{1}{\dfrac{\Delta\rho}{\rho'}\cdot\dfrac{p_s'\Delta T D_c}{4\sigma} - \dfrac{\alpha D_c}{2\lambda'}}, \tag{2.18}$$

or

$$\frac{D_c}{l} = \frac{4|Ku|}{\eta\cdot|Ja|}, \tag{2.19}$$

where

$$|Ku| = \frac{c_p'\rho'^2 gl}{p_s'\rho''\Delta h}, \tag{2.20}$$

and η is defined by

$$\eta = 1 - \frac{2|Nu|\cdot|Ku|}{|Ja|} = 1 - \frac{2|Nu|^2|Ku|}{|Re|\cdot|Pr|} > 0, \tag{2.21}$$

where

$$|Nu| = \frac{al}{\lambda'} = \frac{ql}{\lambda'\Delta T} = \frac{|Re|\cdot|Pr|}{|Ja|}. \tag{2.22}$$

In uniformly superheated liquid there is $\eta = 1$; the greater the heat flux the smaller is η.

Substitution of (2.19) into (2.16) yields

$$|Ja|^2 = \frac{4}{C_\alpha\eta}|Pr|^{\frac{5}{3}}|Ku|\frac{|Re|}{|Ja|} - \frac{\pi}{8C_tC_\alpha C_\beta}\left(\frac{\pi C_j}{6C_\beta}\right)^{\frac{1}{3}}|Ar|^{-\frac{1}{3}} \times (nl^2)^{\frac{1}{3}}|Re|^{\frac{2}{3}}|Pr|^{\frac{2}{3}}. \tag{2.23}$$

Assuming for the moment that both η and n do not depend upon operating conditions one can easily constate that the usually found in experiments relationship $|Re| \sim |Ja|^3$ fulfils the equation (2.23). Namely on substituting

$$|Re| = x^3|Ja|^3 \tag{2.24}$$

into (2.23) one obtains an algebraic cubic equation

$$x^3 - Bx^2 = C, \tag{2.25}$$

where

$$\left.\begin{aligned} B &= \frac{\pi\eta}{32C_nC_t}\left(\frac{\pi C_j}{6C_\beta}\right)^{\frac{1}{3}}\frac{(nl^2)^{\frac{1}{3}}}{|Ku|\cdot|Pr|\cdot|Ar|^{\frac{1}{3}}},\\ C &= \frac{C_\alpha\eta}{4|Ku|\cdot|Pr|^{\frac{5}{3}}}.\end{aligned}\right\} \tag{2.26}$$

The solution may be expressed in terms of the quantity

$$m = \left(\frac{48C_nC_t}{\pi}\right)^3\cdot\frac{6C_\alpha C_\beta}{\pi C_f}\cdot\frac{1}{\eta^2}\cdot\frac{|Ku|^2|Ar|\cdot|Pr|^{\frac{4}{3}}}{nl^2} = \frac{C}{2}\left(\frac{3}{B}\right)^3, \tag{2.27}$$

namely

$$x = \frac{B}{3}\cdot M(m); \tag{2.28}$$

the values of the function $M(m)$ are gathered in Table 1.

Table 1

m	M
0	3·0000
0·1	3·0429
0·2	3·0833
0·5	3·1915
1·0	3·3425
2·0	3·5793
5·0	4·0613
10	4·5803
20	5·2595
50	6·4595
100	7·6570
200	9·1776
500	11·815
1000	14·427
2000	17·670
5000	23·339

For $m \ll 1$, and strictly for $m = 0$ one obtains $M = 3$, and $x = B$. For $m \gg 1$ the result is $M \approx (2m)^{\frac{1}{3}}$, and this gives $x = C^{\frac{1}{3}}$. With this result using (2.22) the equation

$$\frac{|Nu|}{|Re|^{\frac{2}{3}}} = \frac{\pi\eta}{96C_nC_t}\left(\frac{\pi C_j}{6C_\beta}\right)^{\frac{1}{3}}\frac{(nl^2)^{\frac{1}{3}}}{|Ku|\cdot|Ar|^{\frac{1}{3}}}M(m) \tag{2.29}$$

is obtained from (2.24). For $m = 0$ or $m \ll 1$ the above relation is simplified to give

$$\frac{|Nu|}{|Re|^{\frac{2}{3}}} = \frac{\pi\eta}{32C_nC_t}\left(\frac{\pi C_j}{6C_\beta}\right)^{\frac{1}{3}} \frac{(nl^2)^{\frac{1}{3}}}{|Ku|\cdot|Ar|^{\frac{1}{3}}}, \tag{2.30}$$

and for $m \gg 1$ the form

$$\frac{|Nu|}{|Re|^{\frac{2}{3}}} = \left(\frac{C_\alpha}{4}\right)^{\frac{1}{3}} \frac{\eta^{\frac{1}{3}}}{|Ku|^{\frac{1}{3}}}|Pr|^{\frac{4}{9}} \tag{2.31}$$

is valid.

The first simplified formula (2.30) seems to be applicable in the region of discrete bubbles, because in this case the number of bubble columns n is equal to the number of active nucleation sites, and may be therefore considerably great thus causing smaller values of the quantity m. Since the number of activated nuclei is a function of liquid superheat ΔT, or the Jakob number $|Ja|$, and may be assumed as a power function, therefore

$$n \sim |Ja|^r, r > 1, \tag{2.32}$$

which substituted into (2.30) yields

$$|Nu| \sim |Re|^{(2+r)/(3+r)}\,\eta^{3/(3+r)}. \tag{2.33}$$

The value of η may be in this case assumed safely as a constant, because the discrete bubbles region occurs for lower heat fluxes e.g. for water at atmospheric pressure (experiments of Cichelli and Bonilla in the range of $800 < |Re| < 4000$) the value of η varies from 0·887 to 0·934. Since the number of nucleation sites depends upon surface roughness, the formula (2·29) shows that different boiling curves may be obtained for different states of the surface.

In the vapour mushroom region the number of vapour paths is much smaller than the number of nuclei. The value of n is therefore unknown, with the only evidence that it is small. Therefore the quantity m is great, and in this case the second simplified formula (2.31) is more applicable. This formula, however, does not contain the value of n. Since the vapour mushroom region occurs at higher heat fluxes, the value of η may be somewhat more variable than in the previously discussed case. Substitution of (2.21) into (2.31) yields

$$\frac{|Nu|}{|Re|^{\frac{2}{3}}} = \left(\frac{C_\alpha}{4}\right)^{\frac{1}{3}} \frac{|Pr|^{\frac{4}{9}}}{|Ku|^{\frac{1}{3}}}\left[1 - \frac{2|Ku|\cdot|Nu|^2}{|Pr|\cdot|Re|}\right]^{\frac{1}{3}}. \tag{2.34}$$

This is a cubic equation in respect of the Nusselt number and a quadratic equation in respect of the Reynolds number. Solving for $|Re|$ one obtains

$$|Re| = \frac{|Ku|\cdot|Nu|^2}{|Pr|}\left[1 + \sqrt{\left(1 + \frac{4}{C_\alpha}\frac{|Pr|^{\frac{2}{3}}}{|Ku|\cdot|Nu|}\right)}\right]. \tag{2.35}$$

Thus for high Nusselt number values the relation $|Nu| \sim |Re|^{\frac{1}{2}}$ is obtained, i.e. the boiling curve is flatter for higher heat fluxes. If however $|Ku|$ is small, and $|Nu|$ not very great, the value of η can be regarded as constant, and therefore one obtains

$$\frac{|Nu|}{|Re|^{\frac{2}{3}}} = C_0\frac{|Pr|^{\frac{4}{9}}}{|Ku|^{\frac{1}{3}}},\; C_0 = \left(\frac{C_\alpha}{4}\right)^{\frac{1}{3}}. \tag{2.36}$$

This equation contains only one empirical constant.

Simple form of equation (2.36) promises the possibility of simple derivation of it. In truth for $\eta = 1$ the equation (2.36) may be written in this way

$$q = \frac{\lambda'\alpha_D\Delta T^2}{\rho''\Delta hwD_c}, \tag{2.37}$$

wherein equations (1.6), (2.19) and (2.22) have been used. It can be recognized at once that the above formula represents the conduction heat flux at the wall, namely

$$q = \lambda'\left(-\frac{dT}{dy}\right)_{y=0} = (q'')_{y=0}. \tag{2.38}$$

In these conditions, i.e. for $y = 0$, the value

$D(0) = D_c$ is really very small, wherefore the equation (2.3) yields

$$\frac{\mathrm{d}T}{\mathrm{d}y} = \frac{\alpha_D}{\rho''\Delta h w} \cdot \left(\frac{\mathrm{d}\vartheta^2}{\vartheta D}\right)_{y=0}. \qquad (2.39)$$

Replacing $(-\mathrm{d}\vartheta^2/\mathrm{d}D)_{y=0}$ by $\Delta T^2/D_c$ and substituting into (2.39) one obtains

$$\left(-\frac{\mathrm{d}T}{\mathrm{d}y}\right)_{y=0} = \frac{\alpha_D \Delta T^2}{\rho''\Delta h w D_c}, \qquad (2.40)$$

which substituted into (2.38) gives finally (2.37).

The above derivation bases on the assumption that the heat flux is directed from the wall towards the liquid only. This is connected with the very often discussed question whether the latent heat transport from the wall is important or not important. Most opinions are that at low heat fluxes the latent heat flux is very small, but near the maximum critical flux it is predominant. Such statements are based on the assumption that the latent heat flux at the wall is defined by the entire volume of departing bubbles. But in truth during the production and growth of these bubbles heat is transferred from the wall to the liquid, and not to the vapour which in the opposite case were superheated. Therefore the question of percentage of the latent heat transport seems to be physically unessential and strictly conventional.

3. COMPARISON WITH EXPERIMENTAL DATA

Although the pool boiling experimental data are very abundant, also for the geometry of interest (flat horizontal plate), there appears a lack of evidence of certain important factors, which should be determined in the course of experiment.

The first point concerns the microgeometry of the heating surface. The complete description of a clean metallic surface should contain: (1) the mean-square roughness R_{ms}, (2) the maximum roughness R_1, and (3) the statistical distribution of the cavities, i.e. $n_i(R_i)$. The knowledge of these factors in experiments with clean fluid and clean surface shall allow the quantitative verification of the formulae (2.29) or (2.30). As yet the influence of roughness has been studied in a very small number of experimental papers, and the surface roughness has been described either by the kind of machining or at least by the mean-square roughness.

The results of experiments on clean surfaces and fluids may throw light on the physical nature of the process. On comparing these results with the data of experiments carried out in technical conditions, data for surface microgeometry due to contamination and aging may be deduced.

The second point of importance is connected with the known effect of hysteresis in pool boiling. For smooth surfaces the incipience of nucleate boiling occurs usually at higher superheats than for rough surfaces. In experiments with increasing heat flux the superheat rises sometimes very little or even decreases once nucleate boiling has begun. Therefore the boiling curve for increasing heat flux is steeper than that for decreasing heat flux, and only the latter is reproducible. Therefore the comparison of the theory can be made for decreasing heat flux boiling curves only. In these cases the relationship $\alpha \sim q^{\frac{2}{3}}$ has been stated beyond doubt.

The formula (2.36) has been compared with the data of Cichelli and Bonilla [4] for water at

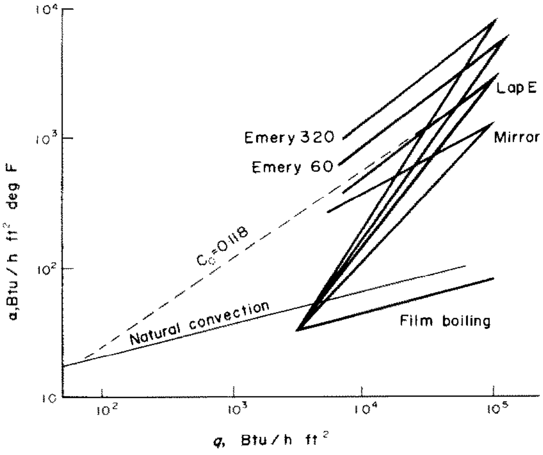

FIG. 1.

saturation temperature between 100 and 250°C. Good agreement has been obtained for $C_0 = 0{\cdot}118$. The same value has been used to draw a line in Fig. 1, based on experiments of Berenson [5] on pentane. These experiments have been carried out to show the influence of surface roughness, and therefore the formula (2.36) is strictly not applicable for the case, although the theory agrees with certain average experimental values. The next step was to compare the formula (2.36) with liquid metals data. Taking as a base the experimental results of Subbotin, Sorokin, Ovetchkin and Kudryavtzev [6] obtained in boiling of sodium and cesium on flat horizontal plates the values of $C_0 = 0{\cdot}148$–$0{\cdot}163$ have been obtained for sodium at $T_s = 800$–900°C, and $C_0 = 0{\cdot}152$–$0{\cdot}188$ for cesium at $T_s = 500$–800°C. It is thus seen that the obtained values of the constant C_0 are of the same order for organic fluids as for liquid metals.

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THÉORIE À TROIS COMPOSANTES DE L'ÉBULLITION NUCLÉÉE EN RÉSERVOIR

Résumé—On améliore dans cet article la théorie à trois composantes du transfert thermique dans l'ébullition proposée antérieurement. Les trois composantes du flux thermique sont: le flux de chaleur latente transportée, le flux thermique de conduction moléculaire et la convection turbulente dans le liquide. Contrairement à l'étude précédente, l'équation de transfert thermique est supposée valable dans la couche limite. Le résultat général de l'intégration et ses formes simplifiées sont discutés. Une de ces formes est obtenue d'une manière simplifiée en tenant compte du gradient de température à la paroi. Cette forme contient uniquement une constante empirique de l'ordre de 0,12–0,18 pour l'eau et les métaux liquides.

VERBESSERTE "DREI-KOMPONENTEN"-THEORIE FÜR DAS BLASENSIEDEN BEI FREIER KONVEKTION

Zusammenfassung—Eine früher vorgeschlagene "Drei-Komponenten"-Theorie für den Wärmeübergang beim Sieden wird in dieser Arbeit weiter entwickelt. Die drei Komponenten des Wärmestromes sind: die Wärmemenge durch Verdampfung; die Wärmemenge durch molekulare Wärmeleitung und die turbulente Konvektion in der Flüssigkeit. Im Gegensatz zu früheren Studien wird angenommen, dass die Wärmeübergangsgleichung auch in der Grenzschicht gültig sei. Das allgemeine Ergebnis der Integration und seiner besonderen vereinfachten Formen werden diskutiert. Eine dieser ursprünglichen Formen wird unter Berücksichtigung des Temperaturgradienten an der Wand in vereinfachter Weise abgeleitet. Diese Form enthält nur eine empirische Konstante von der Grösse 0·12–0·18 für Wasser und Flüssigmetalle.

УСОВЕРШЕНСТВОВАННАЯ «ТРЕХКОМПОНЕНТНАЯ» ТЕОРИЯ ПУЗЫРЬКОВОГО КИПЕНИЯ В БОЛЬШОМ ОБЪЕМЕ

Аннотация—В данной работе проводится усовершенствование предложенной ранее «трехкомпонентнои» модели теплообмена при кипении. Тепловой поток состоит из следующих трех компонентов: скрытой теплоты переноса, молекулярной теплоты теплопроводности и турбулентной конвекции в жидкости. В противоположность предыдущему исследованию предполагается, что уравнение теплообмена справедливо также для пограничного слоя. Обсуждаются обшие результаты интегрирования, а также отдельные упрощенные формы. Одно из этих очень простых уравнений выводится упрощенным методом с учетом градиента температуры на стенке. Это уравнение срдержит только одну эмпирическую постоянную порядка 0,12–0,18 для воды и жидких металлов.